Evaluation of Current Crack Width Calculation Methods According to Eurocode 2 and *fib* Model Code 2010

Reignard Tan[1,2](✉), Max A.N. Hendriks[1,3](✉), and Terje Kanstad[1](✉)

[1] Department of Structural Engineering, Norwegian University of Science and Technology, Rich. Birkelandsvei 1A, 7491 Trondheim, Norway
reignard.tan@multiconsult.no, terje.kanstad@ntnu.no
[2] Multiconsult ASA, Nedre Skøyen Vei 2, 0276 Oslo, Norway
[3] Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands
max.hendriks@ntnu.no

Abstract. The background theory for the crack width calculation methods according to Eurocode 2 and *fib* Model Code 2010 is discussed to evaluate the applicability for the more general case of relatively thick beams, slabs and shells. Essentially, the formulas originate from the maximum transfer length and the difference in steel and concrete strains over this length. It is shown that the formulas are based on both a slip and a no-slip theory, two theories using exactly opposite assumptions. The slip theory assumes that a physical slip occurs in the interface between concrete and steel and, also, that plane sections remain plane. The no-slip theory assumes that no physical slip occurs between concrete and steel and, thus, that plane sections no longer remain plane. The theories were merged pragmatically in an attempt to describe the physical reality related to cracking. This resulted in a formula for the transfer length composed by two linear terms. Such a formulation, however, leads to inconsistencies that opposes the basic principles in solid mechanics. It is argued that these inconsistencies limits the application for the more general case. The observations in this paper suggests that a more robust and consistent calculation method should be formulated. A possible way is by improving the bond assumptions in the interface between concrete and steel, and thoroughly studying the geometry and configuration of cracks experimentally and theoretically.

Keywords: Crack width · Calculation · *fib* Model Code 2010 · Eurocode 2

1 Introduction

The crack width calculation methods according to Eurocode 2 (CEN 2004) and *fib* Model Code 2010 (*fib* 2013) are often straightforward in use for relatively simple geometries, e.g. *regular* beam and slab dimensions. The question, however, is how applicable these formulas are for the more general case. This question is discussed by revisiting the background theory and the origin of the formulas. The discussion is limited to the case for uniaxial tension. It will be shown that the theories used for

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deriving the formulas are based on conceptually two completely different approaches, one being the slip theory and the other being the no-slip theory (Beeby 1979). Both will be elaborated first in the following sections. Next, the inconsistencies of the current formulas will be discussed and a direction for improvement will be suggested.

2 Slip Theory

Basic concept. Saliger (1936) was the first to present the basic concepts in the slip theory. It is conceptually shown in Fig. 1(a). When a crack forms at $x = l_{t\tau}$, strains which once were evenly distributed between concrete and steel will be completely localized in the steel alone at a crack. The redistribution of stresses gradually leads to different strains in concrete and steel ($\varepsilon_c \neq \varepsilon_s$), thus causing a physical *slip* to occur, see Fig. 1(a). The slip will decrease with increasing distance from the crack, owing to the presence of bond stresses in the interface between concrete and steel, see Fig. 1(b). At a certain distance or transfer length $l_{t\tau}$ from the crack tip, the slip may vanish completely such that strain compatibility is restored between concrete and steel again ($\varepsilon_c = \varepsilon_s$), see strain distribution at different sections in Fig. 1(a). Furthermore, plane concrete sections remain plane in the slip theory, which implies that the slip is uniform over the cover.

Considering the local compatibility and the local equilibrium of an arbitrary section within the transfer length, as well as using linear elastic materials laws for concrete and steel, leads to a second order differential equation for the slip. The *crack formation stage* and the *stabilized cracking stage* yields two different sets of boundary conditions for the slip and the first derivative of the slip. The differential equation for the slip can now be solved if the bond stress distribution is known. Finally, the crack width is obtained if the designated transfer length $l_{t\tau}$ is known.

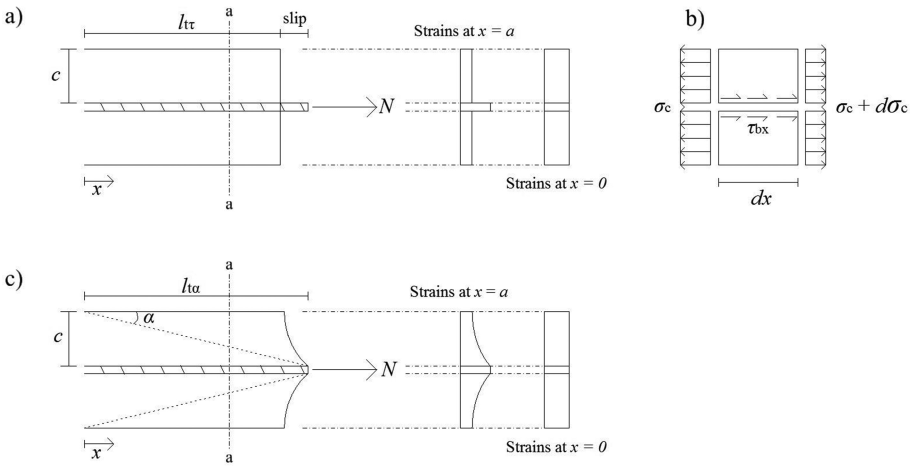

Fig. 1. (a) Slip theory. (b) Local equilibrium concrete in slip theory. (c) No-slip theory

Transfer length. Considering the local equilibrium in Fig. 1(b) yields a first order differential equation for the concrete stresses. It can be solved by using the fact that concrete stresses vanish at a crack and acknowledging that concrete stresses never can exceed a certain tensile strength f_{ctm} at the end of the transfer length (i.e. at $x = 0$). Furthermore, by assuming a constant bond stress distribution τ_{bm} results in the following relation for the transfer length according to the slip theory

$$l_{t\tau} = \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho_s} \tag{1}$$

where ϕ is the steel bar diameter and $\rho_s = A_s/A_c$ is the reinforcement ratio.

3 No-Slip Theory

The no-slip theory was first introduced by Base et al. (Base et al. 1966) and uses assumptions exactly opposite to that in the slip-theory, by discarding the occurrence of a physical slip in the interface between concrete and steel. Hence, perfect bond is assumed such that strain compatibility between the materials is ensured in the vicinity of the steel interface, but deviates elsewhere throughout the cover, see Fig. 1(c). A necessary transfer length $l_{t\alpha}$ from the deformed cracked face is needed to obtain a uniform strain distribution over the cover again. This further implies that strain compatibility is restored at the end of the transfer length, since no slip can occur at the bar level. The transfer length is assumed to be proportional to the cover depth, c, in accordance with a "traditional engineering rule", assuming that the concentrated stresses from the steel in a crack spreads with a fixed angle α. Thus, the transfer length according to the no-slip theory can be expressed as

$$l_{t\alpha} = k_\alpha c \tag{2}$$

In contrary to the slip theory, plane sections no longer remain plane over the whole transfer length in the no-slip theory, which implies that the slip varies over the cover. Finally, the crack width is obtained by multiplying the mean steel strains, provided that these are known, with the transfer length $l_{t\alpha}$.

4 Semi-Empirical Models According to Eurocode 2 and *fib* Model Code 2010

Characteristic crack width. The general relation for calculating the *characteristic* crack width according to Eurocode 2 and *fib* Model Code 2010 can be derived by considering the tensile behavior of a steel reinforced concrete bar (Balász 2010). Figure 2(a) shows an assumed strain distribution for both concrete and steel over the bar length, where it is noticed that the bar behaves according to the assumptions in the slip theory, i.e. plane sections remain plane and strains vary due to the presence of bond stresses. Steel strains in both the crack formation stage and the stabilized cracking stage are also shown. The characteristic crack width is found by considering the mean strains (ε_{sm}, ε_{cm}) and the deformation compatibility over the maximum crack distance $S_{r,max} = 2l_{t,max}$ in Fig. 2(a) and (b) as

$$w_k = S_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{3}$$

Transfer length. The transfer length can be determined by either using the slip or no-slip theory. Both theories have proven to be satisfactory in separate cases, but there is no consensus in the literature which theory is to be preferred, since both represent the physical reality related to cracking only to a certain extent. The idea of slip is physically realistic since concrete cannot elongate to the same extent as steel in tension. However, not taking into account the fact that plane sections no longer remain plane is controversial. The assumption that plane sections no longer remain plane in the no-slip theory will describe the physical reality related to cracking better. However, no mathematical expression is further derived other than using a "traditional engineering rule". Hence, Ferry-Borges (1966) suggested that both theories should be merged, such that the transfer length could be expressed as a linear additive combination of relations (1) and (2)

$$l_t = l_{t\alpha} + l_{t\tau} = k_\alpha c + \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho_s} \tag{4}$$

Assuming that the mean bond stress τ_{bm} is proportional to the mean tensile strength f_{ctm}, relation (4) shows two coefficients in front of c and ϕ/ρ_s. These can be calibrated experimentally, highlighting the semi-empirical nature of the modeling approach. The concept of relation (4) is shown in Fig. 3, where it is noticed that strain compatibility is gradually adjusted over the respective transfer lengths. The $l_{t\alpha}$ term acknowledges that the concrete area adjacent to a crack is *disturbed* and implies that sections here no longer remain plane, which further contributes to increasing the total transfer length caused by slip alone. It can be shown that both the Eurocode 2 and the *fib* Model Code 2010 adopted this additive concept. The main difference between them is that Eurocode 2 multiplies relation (4) with a factor of 1.7, which provides the 95%-quantile for the statistical variation of a crack width when assuming a log-normal distribution (CEB 1985). This leads to crack widths that generally are larger than those obtained by the *fib* Model Code 2010.

Mean strains. Mean concrete and steel strains (ε_{cm}, ε_{sm}) can in a simplified manner be derived by considering the strain distribution for the pulled bar in Fig. 2(a), and the equilibrium states corresponding to *State I* and *State II* in reinforced concrete structures. Stresses in the steel right before a crack forms is given by relation (5) below, which can be derived by calculating the sectional response in State I and assuming linear elastic material laws for both concrete and steel, where $\alpha_E = E_s/E_c$. Next, relation (6) provides steel strains in a crack immediately after cracking, or more generally, the steel strains in cracks during the crack formation stage. Relation (7) conveniently expresses the steel strains in a crack for additional increases in loads.

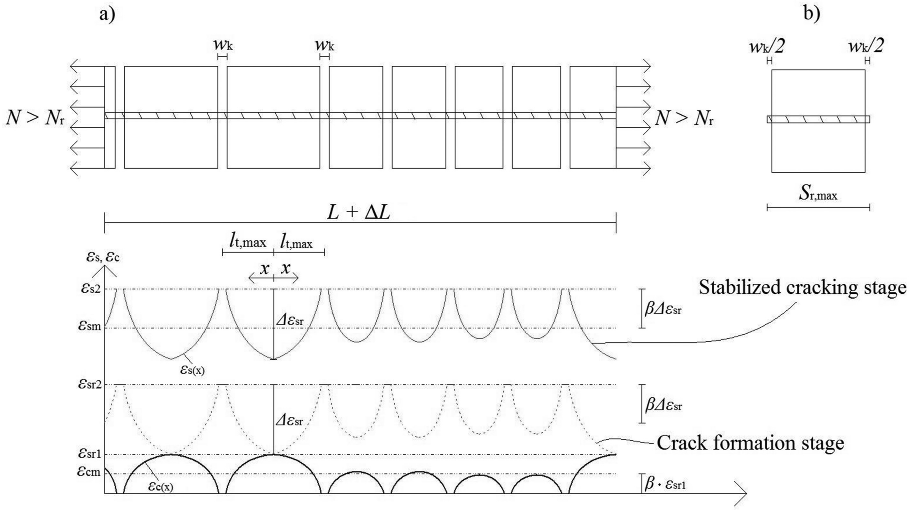

Fig. 2. (a) Strain distribution (b) Cracked RC portion

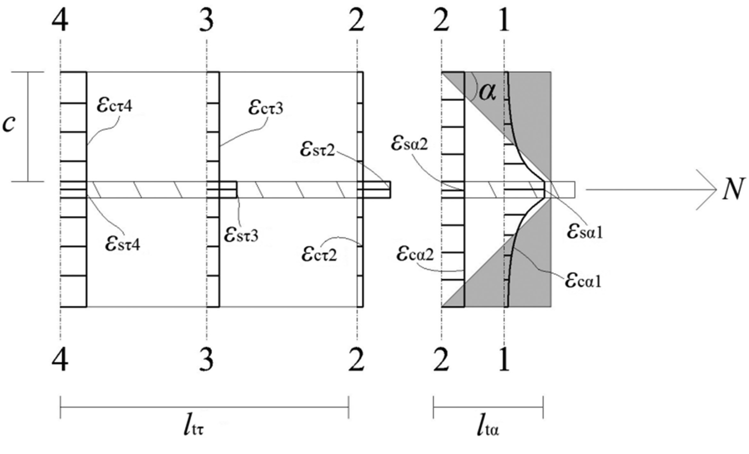

Fig. 3. Strain distribution at different sections over the transfer length

Furthermore, acknowledging that mean steel strains ε_{sm} during the stabilized cracking stage and that mean concrete strains ε_{cm} are as given in Fig. 2(a), as well as acknowledging that $\Delta\varepsilon_{sr} = \varepsilon_{sr2} - \varepsilon_{sr1},$ it can be shown that the strain difference can be expressed as $\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{s2} - \beta\varepsilon_{sr2}$. Finally, inserting relations (6) and (7) in the latter expression results in relation (8), which is the same expression given in Eurocode 2 and *fib* Model Code 2010. Figure 2(a) shows that the factor β is an integration constant and is similar for both the concrete and steel strain distribution. This factor is determined experimentally.

$$\text{Crack stress} \quad \sigma_{sr} = \frac{f_{ctm}}{\rho_s}(1 + \alpha_E \rho_s) \tag{5}$$

$$\text{Steel strains (crack formation stage)} \quad \varepsilon_{sr2} = \frac{\sigma_{sr}}{E_s} \tag{6}$$

$$\text{Steel strains (Stabilized cracking stage)} \quad \varepsilon_{s2} = \frac{\sigma_s}{E_s} \tag{7}$$

$$\text{Strain difference} \quad \varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - \beta\sigma_{sr}}{E_s} \tag{8}$$

5 Discussion

Before discussing the background theory, one should bear in mind that the main focus while developing these formulas was keeping crack width calculations simple and practical, yet with an adequate precision to provide reasonable calculated crack widths in conjunction with measured values from experiments (CEB 1985).

The formulas generally tend to yield good results for relatively *small* specimens, e.g. small beams and slabs. When the geometrical dimensions increase, however, such as the cross section height and the cover, the formulas tend to yield inconsistent results and rather to the conservative side (Rospars and Chauvel 2014). This leads to design cases where the Serviceability Limit State become unfavorably governing, which consequently yields unusual high reinforcement ratios.

The problem seems to be related to the cover term in relation (4). Several authors (e.g. Yannopoulos 1989) have shown that crack widths vary through the cover, being negligibly small at the level of the steel bar before increasing towards the concrete surface. Hence, another interpretation of the inclusion of the cover term may be that the formulas acknowledges the variation of crack widths through the cover, since the no-slip theory in fact accounts for that plane sections no longer remain plane in the concrete area adjacent to a crack, see Fig. 3. Also, it has been seen experimentally that the cover indeed influences the crack distance (Caldentey et al. 2013). Some authors (Gergely and Lutz 1968) argue that the cover term should be dominating, while some (Beeby 2004) even claim that the transfer length should be a function of the cover only. Despite these facts, it cannot be ignored that the inclusion of the cover term clearly violates the equilibrium in relation (1). Debernardi and Taliano (2016) proposes a transfer length without the cover term based on the exact same argument, i.e. that equilibrium at the end of the transfer length is violated for a given mean tensile strength and mean bond stress.

6 Inconsistensies

Relation (4) becomes somehow artificial and peculiar from a structural engineering point of view. The merging of two theories using exactly opposite assumptions is ambivalent. This can be shown by considering the sectional strain distribution at different locations over the composed transfer lengths in Fig. 3. It is noticed that strain compatibility is ensured at the end of each transfer length, which conforms to the assumptions in the slip and no-slip theory. This implies that Sect. 2.2 actually experiences a compatibility and incompatibility in concrete and steel strains, occurring at the same time. This also implies that strain compatibility occurs twice within the composed transfer length. Such inconsistencies, including the fact that equilibrium is violated at the end of the transfer length, opposes the basic principles in solid mechanics and which further limits a generalization of the formulas.

Another questionable observation is, that relation (8) is derived by using the same integration constant β for the mean steel and concrete strains, which cannot be further justified other than it can be obtained empirically.

The notion that the empirical constants adjusting the formulas most likely are based on test results from relatively small test specimens, provides a further explanation for the limited applicability of the crack width formulas. The question of which of the inconsistencies is most accountable for limiting the generalization is in this respect of secondary importance.

7 Suggestion to Improvements

The discussion above suggests first and foremost, that a more robust and consistent calculation method which conforms better with the physical reality related to cracking should be formulated. The second is that this should not be done by simply merging different theories that only to a certain extent describe the physical reality related to cracking. An approach to a more consistent formulation can be investigating how bond in the interface between concrete and steel influences cracking. This seems to have lacked attention when developing the current formulas.

The significance of bond can be elucidated by considering the two first distinct stages in a local bond slip curve. Imagine that the embedded steel bar in Fig. 1(c) is pulled with a force. If the force is relatively small, the chemical bond or rather the adhesion will be activated, and ensures that strains in the concrete are localized in the vicinity of the steel bar. No slip between steel and the surrounding concrete will occur if the localized strains are within the elastic range. Upon further loading, the adhesion eventually breaks down and the mechanical bond, mainly due to the interaction between bar lugs and the surrounding concrete, takes over by still ensuring strain localization near the steel. However, if the localized strains now exceed the tensile strength of concrete, small internal cracks arise and propagate towards the loaded surface (or the primary crack), see Fig. 4. The formation of internal cracks can in this case be considered analogous to the physical occurrence of slip. Goto (1971) discovered the phenomenon first and is now commonly accepted in the literature. These allegations also conform to the observations drawn by Tammo and Lundgren (2009).

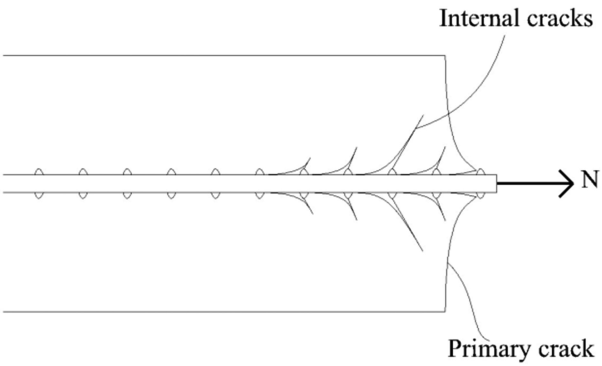

Fig. 4. Internal cracking due to mechanical bond

8 Conclusion

The theoretical background for the crack width formulas according to Eurocode 2 and *fib* Model Code 2010 has been discussed. The transfer length was obtained by simply merging the slip and no-slip theory, since neither alone fully could describe the physical reality related to cracking. This resulted in a formula for the transfer length composed by two linear terms. The theories, however, are based on assumptions completely opposite to one another. Such a formulation leads to inconsistencies that opposes the basic principles in solid mechanics and which further limits a generalization of the formulas. It is suggested that a more consistent formulation can be based on an approach that improves the bond assumptions in the interface between concrete and steel. The authors of this paper are currently investigating such an approach.

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