Reliability Assessments of Large Reinforced Concrete Structures Using Non-linear Finite Element Analyses: Challenges and Solutions

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Abstract. The use of non-linear finite element analyses for reliability assessments of reinforced concrete structures has gained much attention during the last decade, particularly with the introduction of semi-probabilistic methods in *fib* Model Code 2010. In a recent PhD project, the topic has been elaborated on with a special focus on the applicability to large concrete structures like dams and offshore platforms. Such structures usually require the use of relatively large solid finite elements and large load steps in order to reduce the computational cost. In this paper, the main findings from the project, including a proper material model for concrete, quantification of the modelling uncertainty and treatment of uncertainties from different sources, will be discussed. Unanswered questions that has been raised during the project will be highlighted.

Keywords: Analysis and design · Numerical modelling · Large concrete structures · Non-linear finite element analysis · Modelling uncertainty · Structural reliability · Global resistance factor method

1 Introduction

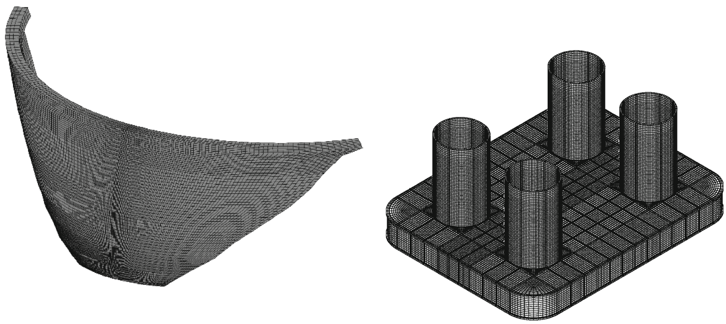
In the last decade, the use of non-linear finite element analyses (NLFEA) for reliability assessments of reinforced concrete structures has gained much attention. In particular after the introduction of semi-probabilistic methods in the *fib* Model Code 2010 (*fib* 2013). When such methods are applied, the results from NLFEA are treated as results from virtual experiments, giving an estimate of the load carrying capacity of the structure in question. Demonstrations of use can be found in the literature (e.g. Schlune et al. 2012, Cervenka 2013, Blomfors et al. 2016). In order to provide reliable results, the analyst needs a good strategy for obtaining results from NLFEA, a *solution strategy*, and a proper method for quantification and treatment of physical uncertainties and modelling uncertainties. This paper summarizes the main findings from an

D.A. Hordijk and M. Luković (eds.), *High Tech Concrete: Where Technology and Engineering Meet*, DOI 10.1007/978-3-319-59471-2_193

industrial PhD-project where these topics have been studied with special focus on the application to the design of large concrete shell structures (Engen et al. 2014).

2 Solution Strategy for NLFEA of Large Concrete Structures

Finite element analyses of large reinforced concrete shell structures like e.g. dams or offshore concrete structures as shown in Fig. 1, normally require a large number of finite elements where the elements are relatively large in order to reduce the computational cost. The failure mode might be due to a combination of different sectional forces, and can be difficult or even impossible to predict before the analysis is performed. Traditionally, shell elements are preferred due their efficiency in modelling and computation, but usually such elements require additional capacity control since the interaction between in-plane forces and moments, and transverse shear forces is usually not accounted for in the NLFEA. The use of solid finite elements and a three dimensional material model for concrete thus seems most applicable, since such an approach in principle can predict any type or combination of failure modes. Solid finite elements also allow the analyst to model the geometry and load application more or less exact, and ensures correct modelling of the stiffness in structural joints, and are thus preferred in analyses that are basis for the design of large concrete shell structures, as shown in Fig. 1 (Brekke et al. 1994).



(a) (b)

Fig. 1. Examples of large concrete shell structures (Multiconsult ASA). (**a**) Dam Sarvsfossen, (**b**) Offshore concrete structure.

A solution strategy for NLFEA comprises choices regarding force equilibrium, kinematic compatibility and material models. Results from benchmark analyses with two different solution strategies, differing only in terms of the selected material models for concrete, were compared in order to find a material model to apply in the further work (Engen et al. 2014, 2015). The first material model was based on a set of guidelines (Hendriks et al. 2016) and consisted of uniaxial models in compression and tension extended with complementary models taking into account the effects of e.g. confinement and lateral cracking. The second material model was a fully three-dimensional material model for concrete where the effects of a three-dimensional stress state on both capacity and ductility were automatically accounted for (Kotsovos 1979, 1980; Markou and Papadrakakis 2013). The results from this comparative study showed that the second material model predicted results in closer agreement with the experimental results, despite treating the behaviour in tension and compression in a brittle manner. It was thus recommended to shift the attention from a detailed description of the tensile post-cracking behaviour to a rational description of the compressive behaviour, at least when relatively large elements are used and the ultimate limit capacity is sought (Engen et al. 2015). It should be noted that in this work, *large elements* should be interpreted as elements that are large relative to the cross-sectional dimensions, typically two to three elements over the beam height or slab thickness.

The fully three-dimensional material model was further adapted to a fixed, smeared and non-orthogonal cracking framework to facilitate its implementation in a standard finite element software (Engen et al. 2017c). A maximum number of three cracks were allowed per integration point and the model handles both crack opening and closure. Since only the cylinder compressive strength is required, the material model is regarded as applicable to practical engineering problems. The reinforcement was modelled as embedded, fully bonded elements with a bi-linear elastic-plastic material model. The external loads were applied with relatively large constant load increments. Regular Newton-Raphson with line-search was used for the iterative solution of the equilibrium equations, and convergence was controlled with criteria for force and energy. The applicability to large structures has been demonstrated in this work (Engen et al. 2017b, c).

When using large finite elements and relatively large finite load steps, convergence can be hard or even impossible to obtain in load steps where a large number of cracks are initiated. There is thus need for a method for distinguishing load steps with global failure from load steps that simply did not reach convergence. In the present work, two methods were considered: (i) nodal displacement increments visualized as deformed shape and (ii) finding a critical value for the global stiffness degradation given by the *current stiffness parameter* suggested by Bergan et al. (1978).

3 Modelling Uncertainty

Models in engineering analyses are never right or wrong, but can be useful if the modelling uncertainty is properly accounted for. The modelling uncertainty, θ, depends on the mathematical idealization and the numerical solution of the problem at hand, and according to Ditlevsen (1982), is related to the limitation of a possible infinite number of basic variables to a finite number, either for pragmatic reasons, or due to lack of knowledge. The modelling uncertainty thus implicitly includes what is not explicitly considered in the model, i.e. both the known and the unknown unknowns. The modelling uncertainty of the solution strategy, which can be defined as the ratio between the experimentally and numerically obtained capacity, $\theta = R_{\mathrm{exp}}/R_{\mathrm{NLFEA}}$, was studied in 38 benchmark analyses (Engen et al. 2017a). Bayesian inference with a non-informative prior showed that θ could be modelled as a log-normally distributed random variable with a mean 1.10 and a standard deviation 0.12.

The sample of benchmark analyses contained failure modes in the range from fully brittle to fully ductile, characterized by concrete failure or yielding of the reinforcement respectively (Engen et al. 2017a). The sample could have been separated in smaller samples depending on the failure modes, but since the failure mode of large concrete shell structures most likely is due to an interaction between different sectional forces, the collection seems justified. The results indicated that the modelling uncertainty had a higher bias and a higher standard deviation for brittle failure modes compared to ductile failure modes. This can be interpreted in two ways, either (i) the accuracy of the solution strategy depends on the failure mode, or (ii) the variation of the experimental result depends on the uncertainty of the material parameter that dominates the failure mode. By inspection of e.g. the variation of the experimentally obtained shear capacity, ii) is expected to be significant and it was thus concluded that θ includes a contribution from the material uncertainties. This is regarded as a rational explanation to the different values of the modelling uncertainty that have been reported for different types of failure modes in the literature (e.g. JCSS 2001; Schlune et al. 2012).

Figure 2 indicates how the level of knowledge, or the level of control of variation in the experiment, influences the contribution from material uncertainties to the estimated modelling uncertainty. If only the strength class is known, the analyst has no control of the actual variation of the material strength within the specimen, and all of the material uncertainty will be included in the modelling uncertainty. In this extreme case, θ is the only basic variable that carries probabilistic information. If, on the other hand, the analyst has exact knowledge of the variation of the material parameters and the spatial variation within the specimen, a pure modelling uncertainty can be obtained and the material uncertainties should be treated by separate random variables. This extreme case, however, is only theoretical, since one cannot obtain full knowledge about the material uncertainty using non-destructive material testing.

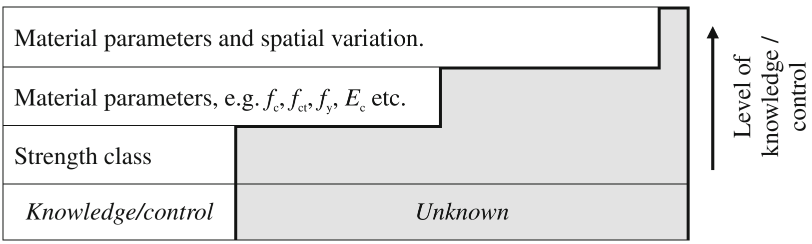


Fig. 2. The influence of the level of knowledge on the contribution from material uncertainties to the modelling uncertainties, indicated by the shaded areas.

The results of this work might be somewhere in between the two extreme cases since the NLFEA were performed using the cylinder strength reported in the literature which was assumed to be an average value of a small number of samples from the same batch as the experiment specimen. It can thus be stated that the within-batch variation is included in the estimate of θ, but that only a part of the between-batch variation is included (Engen et al. 2017a).

The implications of this are unfortunate, since with the present definition of θ, the distillation of the modelling uncertainties is not straight-forward. However, it might also be useful, since one can argue that e.g. the unknown correlation between cylinder compressive strength and tensile strength or Young's modulus is implicitly included in θ since the inference on θ is based on experimental results from concretes of several origins, and possibly a variation of different relations between the material parameters.

If more refined material models are used, possibly requiring more material input parameters, the values of the parameters are often estimated, e.g. by relations between the parameter in question and the cylinder strength, or the NLFEA are calibrated, i.e. the values of the parameters are adjusted in order to obtain numerical results in close agreement with the experimental results. It is important to realize that the uncertainty related to the estimates of additional material parameters adds to the modelling uncertainty, and that such estimates should be done consistently from analysis to analysis in order to obtain a reasonable and quantifiable modelling uncertainty. As soon as the material parameters are adjusted from the *a priori* estimates in order to obtain results in closer agreement with the experimental results, it is believed that uncertainties related to human error might increase. This was not further elaborated on in the present work since all NLFEA were performed using the reported cylinder strengths without calibration to improve the results.

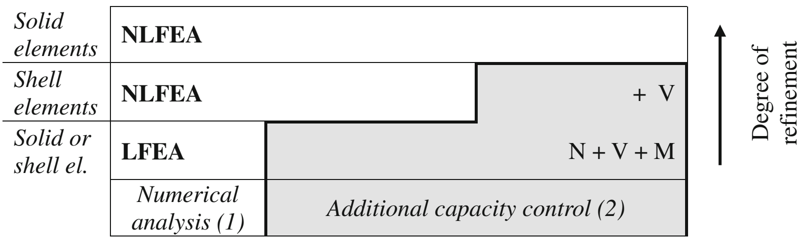


Fig. 3. The effect of model refinement on the sources of modelling uncertainty, adapted and simplified from the work by Plos et al. (2017).

The modelling uncertainty also depends on the refinement of the numerical analysis. By following a multi-level assessment approach (Plos et al. 2017), the degree of refinement can be visualized as shown in Fig. 3. The Figure gives three examples of refinement of the numerical model used for design of concrete structures:

(i) Linear finite element analyses (LFEA) with solid or shell elements and additional sectional capacity control.
(ii) NLFEA with shell elements and additional control of the transverse shear capacity.
(iii) NLFEA with solid elements.

Typically, level (i) is used in regular structural design and levels (ii) and (iii) are only invoked in case of assessments of the design of complex new structures, or assessments of existing structures. Assuming that the modelling uncertainty of the numerical analysis, θ_1, is independent of the modelling uncertainty of the additional capacity control, θ_2, the coefficient of variation of the modelling uncertainty, v_θ, can be roughly estimated as $v_\theta \approx \sqrt{v_{\theta 1}^2 + v_{\theta 2}^2}$. At level (iii) no additional capacity control might be needed, and the modelling uncertainty as quantified by Engen et al. (2017a) can be used directly. At level (ii), the modelling uncertainty of the numerical analysis might be in the order of level (iii), but the additional control of the transverse shear capacity might have a relatively large modelling uncertainty resulting in $v_{\theta \mathrm{ii}} \geq v_{\theta \mathrm{iii}}$. Also, if the LFEA at level (i) was replaced by a NLFEA, the modelling uncertainty of the numerical analysis would change, meaning that the analyst should take into account other uncertainties in the regular design based on the level (i) approach. This simple example illustrates the importance of properly including the modelling uncertainty.

It should be noted that the present project has only considered the part of the modelling uncertainty related to the accuracy of the model, assuming that one specific model, or solution strategy, has been selected. Generally, the modelling uncertainty also includes a contribution from model selection, assuming that any thinkable or unthinkable solution strategy can be selected. If the contribution from model selection was to be included, the effort from improving the numerical model would only briefly be rewarded, since the analyst would be forced to take into account the modelling uncertainty of other and possibly weaker models. Instead, only the accuracy of the model should be considered, and it should be the responsibility of the analyst to either select a solution strategy based on a guideline (e.g. Hendriks et al. 2016) or based on a thorough assessment of the modelling uncertainty (e.g. Engen et al. 2017a, c).

4 Further Challenges

The need for a global failure criterion was only briefly described above and in separate publications (Engen et al. 2017b, c). Such a criterion seems to be most applicable to NLFEA of large statically indeterminate structures with large elements, where a lack of convergence not necessarily is coinciding with global failure. It might be valuable to study further the *current stiffness parameter* or similar indicators, and suggest critical values for which global failure can be assumed to initiate. It could be interesting to see the global failure criterion in relation with the modelling uncertainty.

With the present definition of the modelling uncertainty, it is evident that the material uncertainties contribute to the estimated modelling uncertainty. How large the contribution is, and which parts of the material uncertainties that should be included separately should be investigated further. With a hierarchical modelling approach, the uncertainties can be studied on different levels, e.g. within-batch, between-batch or between-producer. This could be accomplished by studying data from e.g. concrete plants or existing structures, possibly using prior information from the literature (e.g. Rackwitz 1983; Bartlett and MacGregor 1996; JCSS 2001) in a Bayesian framework.

a) Close to linear limit state function.

b) Non-linear limit state function, where two failure modes are clearly interacting.

Fig. 4. Examples of limit state functions with two basic variables z_1 and z_2.

The effect of the failure mode on the applicability of the semi-probabilistic methods is not yet fully understood, and with reference to Fig. 4, two main challenges are put forward:

(i) If two failure modes are interacting, depending on the shape of the limit state function, the linearization of the limit state function which is assumed in e.g. ECOV or the method suggested by Schlune et al. (*fib* 2013; Schlune et al. 2012) might be unconservative.
(ii) If different basic variables are governing in different failure modes, the FORM sensitivity factors of the different variables will change depending on the failure mode, and the resulting partial factors will change.

However, if the variability of the resistance is independent of the failure mode, and the limit state is linear, the results of the semi-probabilistic methods should not depend on the failure mode. In a design situation, the engineer would in most cases not know the failure mode nor the failure load, and should perform several analyses in order to be able to interpret the problem at hand and to study the sensitivity of the results to the input variables to the analysis.

5 Conclusion

Through the last decades, NLFEA have been demonstrated to have a large potential in everyday engineering projects, and further innovation is promoted by the introduction of suitable semi-probabilistic or refined methods in future codes. In a codified framework, and supported by international guidelines, NLFEA can eventually be safely applied during design of concrete structures. One of the main findings from the reported project indicate that at present the modelling uncertainty of NLFEA includes a contribution from the material uncertainties, and that this contribution is usually included twice in reliability assessments, resulting in what is expected to be conservative estimates of the structural capacity. The estimates should always be conservative, however since NLFEA are usually relatively costly, the degree of conservatism should be reduced. Further work aiming at reducing the unintended conservatism is suggested.

Acknowledgements. The work presented in this paper is part of an industrial PhD project funded by Multiconsult ASA and The Research Council of Norway. Morten Engen thanks his supervisors and all colleagues in the Marine Structures Department at Multiconsult for valuable discussions and particularly Per Horn, former Senior Vice President of Multiconsult, for having the courage to initiate the research project. With the aid from Dr. Gerd-Jan Schreppers, CEO of DIANA FEA, the material model discussed in this paper is made available in DIANA v. 10.1.

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